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# EFFICIENT COMPUTATION OF STEADY WATER FLOW WITH WAVES

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**Key words:** Steady Water Waves; Volume-of-Fluid; Compressive Limiter; Multigrid; Defect Correction.

**Summary.** A surface capturing model for steady water flow that can be solved very efficiently is presented. The model contains a high-accuracy water surface discretisation and turbulence; it is solved with a novel linear multigrid technique and defect correction. Results show the accuracy of the model and the fast convergence of the solver.

## 1 INTRODUCTION

Numerical simulation of steady water flow with gravity waves is of major importance in marine design. For example, accurate prediction of a ship's wave pattern and the interaction of the waves with the viscous flow near the ship hull enables the minimisation of the ship's drag in the design phase, reducing operating cost and environmental impact.

To model the water surface, two techniques exist: surface fitting and surface capturing[3]. For surface fitting, the grid is deformed during the computation, such that its upper boundary coincides with the water surface. This is a mature technique that gives accurate solutions and is computationally efficient. But it is not flexible: the grid deformation does not allow complex-shaped ship hulls. For surface capturing, the grid is not deformed but the water surface moves through the grid. Examples are the volume-of-fluid and level-set technique. These can handle arbitrary hull shapes.

A limiting factor for surface capturing is the solution speed: as opposed to surface fitting, fast solution techniques are not readily available. Usually, the location of the interface is reconstructed and boundary conditions are imposed there. It is this reconstruction that makes efficient solution very difficult; time stepping to convergence is the usual, costly solution technique.

We show that a surface capturing discretisation, including an accurate water surface model and turbulence, can be solved highly efficiently. This is achieved with a volume-of-fluid discretisation which is formulated such, that it can be solved with multigrid. Second-order accuracy is obtained with defect correction.

## 2 FLOW EQUATIONS

We solve the flow both in the water and in the air above it. The flow equations are based on the Reynolds-Averaged Navier-Stokes (RANS) equations; we distinguish between water and air by adding a mass conservation equation for the water. Thus, the complete system consists of nothing but conservation laws, it has no interface reconstruction. Therefore it can be solved well with multigrid. In two dimensions, the system is:

$$
\begin{aligned}
\frac{\partial}{\partial x}\left(p+\rho u^2\right)+\frac{\partial}{\partial y}(\rho u v) &= \frac{\partial}{\partial x}\left((\mu+\mu_T)\,2u_x\right)+\frac{\partial}{\partial y}\left((\mu+\mu_T)\,(u_y+v_x)\right) && (x\text{-mom.}),\\
\frac{\partial}{\partial x}(\rho u v)+\frac{\partial}{\partial y}\left(p+\rho v^2\right) &= \frac{\partial}{\partial x}\left((\mu+\mu_T)\,(u_y+v_x)\right)+\frac{\partial}{\partial y}\left((\mu+\mu_T)\,2v_y\right)-\rho g && (y\text{-mom.}),\\
\frac{\partial}{\partial x}(u)+\frac{\partial}{\partial y}(v) &= 0 && (\text{tot. mass}),\\
\frac{\partial}{\partial x}(u\alpha)+\frac{\partial}{\partial y}(v\alpha) &= 0 && (\text{water mass}).
\end{aligned}
\tag{1}
$$

with $\alpha$ the volume fraction of water. The turbulent viscosity $\mu_T$ is computed with Menter's one-equation turbulence model[4]:

$$
\frac{\partial(\rho u\nu_T)}{\partial x}+\frac{\partial(\rho v\nu_T)}{\partial y}=\frac{\partial}{\partial x}\left((\mu+\mu_T)\,\frac{\partial\nu_T}{\partial x}\right)+\frac{\partial}{\partial y}\left((\mu+\mu_T)\,\frac{\partial\nu_T}{\partial y}\right)+P-D, \tag{2}
$$

with $\nu_T = \mu_T/\rho$. The production $P$ and the dissipation $D$ contain first and second velocity derivatives.

## 3 FLUX DISCRETISATION

The flow equations are discretised with cell-centred finite volumes on structured curvilinear grids. Multigrid is used to solve a first-order accurate discretisation and these solutions are improved with defect correction on a second-order accurate discretisation. For robustness at high Reynolds numbers, the convective and diffusive fluxes are discretised separately. Stable convective fluxes are obtained with a Riemann solver[6]. The diffusive fluxes and the turbulence source term are modelled with central differences.

To ensure stable and monotone solutions, the second-order scheme uses limited reconstruction of the cell face states for the convective fluxes. For most state variables, a standard limiter is used. However, the volume fraction solution is a smeared out discontinuity only. Therefore, we can use a compressive limiter for $\alpha$, like the limited downwind scheme. Such a limiter steepens any gradient into a discontinuity, so it keeps the surface sharp. For unsteady flow, the downwind scheme deforms the surface into a staircase. But in our steady flow, the surface is always parallel to the flow direction, so the surface location is fixed by the velocity field; large stairstep deformations cannot occur. Still, following Ubbink and Issa[5], we use a hybrid limiter that switches to a second-order scheme when the surface is parallel to a cell face normal. This prevents even small stairsteps.

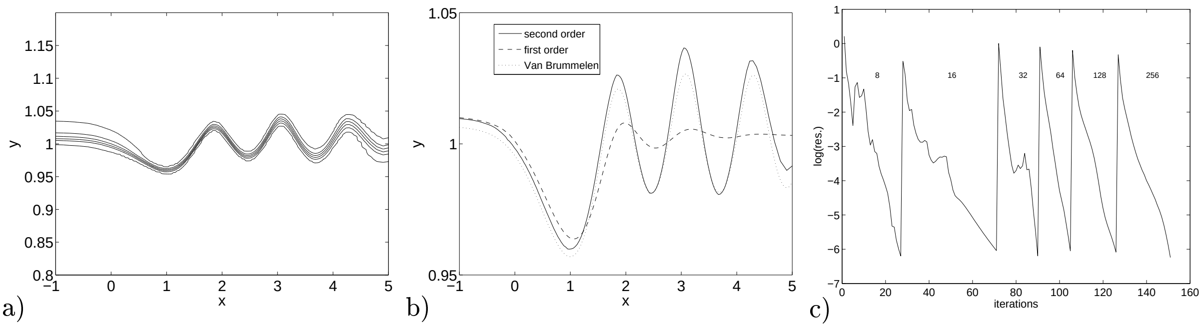


Figure 1: Solution of subcritical channel flow. Second-order volume fraction $\alpha$ (a) (the grid is stretched from $x = 4$, the inflow boundary causes the initial smearing), a comparison of the $\alpha = 0.5$ isolines with the numerical result of Van Brummelen[1] (b), and the residual during the full mutigrid computation (c).

## 4 FAST SOLVER

The flow equations are solved with multigrid and line Gauss-Seidel smoothing. All the equations, including the turbulence model, are solved in a coupled way; this is essential to get the best convergence. Solving this complicated system requires two changes to the standard multigrid technique.

First, the turbulence model makes the line smoothing unstable sometimes. However, this does not occur near a converged solution and when it occurs, the instability can be removed with local damping of the smoother. This damping can be estimated from the convergence of the nonlinear solver in each individual line. Thus, we get highly efficient and stable smoothing.

And second, the turbulent boundary layers and the thin water surface layer cannot be resolved accurately on coarse grids. Therefore, the coarse-grid solutions do not resemble the fine-grid solution well, which results in reduced multigrid convergence. We correct this by changing to linearised coarse grid corrections with Galerkin operators.

We solve the second-order accurate equations with defect correction: the second-order residual is put as a source term in the first-order equations, which are then solved with multigrid. This process converges slowly, but convergence is not necessary as a few defect correction steps already improve a first-order accurate solution to second-order accuracy. With added damping in the air region above the interface, defect correction is stable for our system.

## 5 RESULTS

Results are shown for two 2D channel flows with a bottom bump, as measured by Cahouet[2]. Both are computed on $256 \times 256$ cell grids, with fine cells near the bottom and the free surface. The first test is a subcritical flow, with $Fr = 0.43$ (figure 1). For the first-order solution, the waves damp out quickly. The second-order solution after 8 defect correction steps shows a great improvement: the wave amplitudes are much higher. The compressive scheme makes the water surface, that smears at the inflow boundary, thinner near the first wave and keeps it thin ($4 - 5$ cells). A comparison with the numerical results of Van Brummelen[1] shows excellent agreement.

A supercritical flow ($Fr = 2.05$) is shown in figure 2. Here, the first-order solution captures the shape of the wave, but the second-order solution (5 defect correction steps) is much sharper. The surface is $3 - 4$ cells thick throughout. The agreement with Cahouet is good.

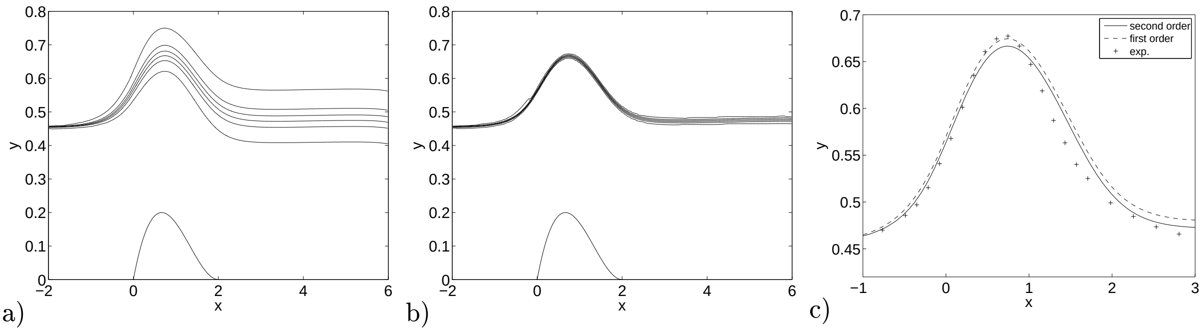


Figure 2: Supercritical channel flow test: the volume fraction for the first-order (a) and second-order accurate solution (b), and a comparison with Cahouet's experiment[2] (c).

The multigrid convergence on 7 grids for the first test case is shown in figure 1c. It is a full multigrid computation: the initial solution on each grid is found by propagating the solution on the next coarser grid. The convergence on the last grids is just as good as for the same problem with laminar flow[6]. The computation time is compared with a solution by line smoothing only: multigrid is about 20 times faster. Defect correction requires a few cycles only, it adds about a third to the first-order computation time.

## 6 CONCLUSION

A VoF model without interface reconstruction is proposed for steady water flow. The model consists of conservation laws only, so it can be solved with multigrid. Linear multigrid gives very fast convergence, even for turbulent flow. High-accuracy solutions and good agreement with experiments are obtained with a combination of defect correction and a compressive discretisation for the volume fraction.

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